# Adaptive Generic Model Control: Dual Composition Control of Distillation

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*Generic model control (GMC) takes care of parametric mismatch for underdamped, closed-loop specification, whereas robust generic model control (RGMC) can handle parametric mismatch for any closed-loop specification. But, neither GMC nor RGMC is capable of compensating for structural mismatch. In this study, adaptive GMC(AGMC) and adaptive RGMC (ARGMC) structures are proposed, and their effectiveness over GMC and RGMC is demonstrated with several examples. AGMC exhibits better performance over ARGMC, GMC, and RGMC in all the cases of no process/model mismatch, parametric mismatch as well as structural mismatch.*

*Distillation adaptive generic model control (DAGMC) structure is also proposed for dual composition control of distillation. Since embedding of distillation state-space model in the basic GMC law is practically impossible, linear and nonlinear models are proposed with adaptation using distillation process data, and DAGMC is applied to two typical nontrivial distillation units. Nonlinear DAGMC exhibited better performance over linear DAGMC.*

## Introduction

The strong nonlinearities of many chemical processes limit the application of model-predictive, linear, multivariable controllers. For the control of nonlinear processes, Economou et al. (1986) developed an internal model control (IMC) approach, which employs nonlinear models in the control strategy. This technique, however, required the user to develop complex numerical inverses to the process model.

Recently, Lee and Sullivan (1988) introduced a generic model control (GMC) that employs a nonlinear process model directly into a control strategy. In a case study, Lee et al. (1989a) have demonstrated that the performance of GMC is superior over both traditional and modern control strategies for a forced circulation evaporator. GMC has been applied successfully to the temperature control of exothermic batch reactor for the startup and subsequent temperature maintenance (Cott and Macchietto, 1989). The process-model-based engineering approach (Cott et al., 1989) is shown to have several advantages over existing approaches, as process optimizers may be incorporated and integrated easily with the GMC-based controllers to provide an effective controller-optimizer system. GMC as the process-model-based control is applied for the pH control of wastewater (Williams et al., 1990).

Lundberg and Bezanson (1990) have shown that GMC lacks robustness for critical and overdamped closed-loop specifications and therefore proposed an enhanced robust GMC by using derivative feedback to take care of process/model mismatch. The enhanced robust GMC (RGMC) structure is similar to the model-predictive control structures—IMC (Garcia and Morari, 1982), MAC (Rouhani and Mehra, 1982), and DMC (Cutler and Ramaker, 1979) in model error compensation, but is distinct in that RGMC estimates and compensates for the error between the process and process model output time derivative.

The real process and process model for control can differ in two ways: one in which the structure of the process model is the same as the real process, but with different parameters—parametric mismatch; the other in which the structure of the process model differs from the real process—structural mismatch. GMC takes care of parametric mismatch for underdamped closed-loop specification, whereas RGMC can handle

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parametric mismatch for any closed-loop specification (that is, underdamped, critically damped, or overdamped). But, neither GMC nor RGMC is capable of compensating for structural mismatch.

Most of the real chemical processes are nonlinear, and some chemical and biochemical processes change behavior over a period of time. Moreover, the fundamental process model equations, which are necessary for GMC, incorporate all the states encountered in the process. In most of the process control systems, very few states are measurable and it is quite difficult to infer the rest of the states from the measured states. In such a case, it is necessary to have a model (linear or nonlinear) that only requires information about the measurable states. This causes a structural mismatch between the process and the model, which cannot be compensated by RGMC.

To keep optimal system performance at all times while controlling such complex systems, on-line adaptation of the process model is necessary, using information of the measurements. In this study, adaptive GMC (AGMC) and adaptive RGMC (ARGMC) structures are proposed, and their effectiveness over GMC and RGMC is demonstrated with several examples. These control structures can be employed for the single-input/single-output control as well as multi-input/multi-output processes.

A distillation adaptive generic model control (DAGMC) structure is also proposed and applied, using linear and nonlinear model equations with adaptation, to two typical nontrivial distillation units: a methanol-water distillation column (Wood and Berry, 1973) and a debutanizer (Gani et al., 1986; Ruiz et al., 1988).

## Generic Model Control

Recently, Lee and Sullivan (1988) have generalized many of the model-based techniques into a generic structure called the generic model control, which allows the incorporation of nonlinear process models directly in the control algorithm. Consider a process described by:

$$\dot{x} = f\ (x,u,t) \tag{1}$$

$$y_m = g\ (x) \tag{2}$$

where $x$ is a state variable, $u$ is the manipulated input variable, and $y_m$ is the output of the process model. In general, $f$ and $g$ are some nonlinear functions.

The generic model control is an optimal control approach to forcing the process output rate to match a reference rate. The reference rate for the process is generated from the set point deviation as:

$$r^* = k_1 (y^* - y) + k_2 \int (y^* - y)\, dt \tag{3}$$

where $r^*$ is the desired process output "rate of change," $y^*$ is the set point, and $k_1$, $k_2$ are the generic control-loop constants. The reference rate is proportional to the distance from the set point and includes integral action to eliminate offset. The control signal is obtained by setting:

$$r^* = \dot{y}_m = \dot{g}\,(x) \tag{4}$$

and solving for the manipulated input, $u$. A major advantage of GMC is that a generalized or "generic" response profile, independent of the particular process, may be specified through choice of $k_1$ and $k_2$.

Equations 3 and 4 can be written in Laplace domain as:

$$\frac{y}{y^*} = \frac{2\xi\tau s + 1}{\tau^2 s^2 + 2\xi\tau s + 1} \tag{5}$$

where $\tau = 1/\sqrt{k_2}$ and $\xi = k_1/2\sqrt{k_2}$. Equation 5 provides the response to set point changes. This system does not yield the same response as a classic second-order system. A tuning map of the normalized response of the system $y/y^*$ vs. normalized time $t/\tau$ was constructed with $\xi$ as a parameter and used to select the generic control-loop constants $k_1$ and $k_2$, which result in the desired shape and speed of response (Lee and Sullivan, 1988). This normalized output response was reported in the form of a figure for the four values of $\xi$. It is not possible to assume any other $\xi$ value than the reported ones without a loss of accuracy. Also, some errors may be introduced while reading that figure. Therefore, the following equations are derived from Eq. 5:

*Underdamped, $\xi < 1$:*

$$\frac{y}{y^*} = 1 - [\exp(-\xi t/\tau)]\{\cos[(t/\tau)(\sqrt{(1-\xi^2)}] - [\xi/(\sqrt{(1-\xi^2)})][\sin((t/\tau)(\sqrt{(1-\xi^2)})]\} \tag{6}$$

*Critically damped, $\xi = 1$:*

$$\frac{y}{y^*} = 1 - [\exp(-t/\tau)](1 - t/\tau) \tag{7}$$

*Overdamped, $\xi > 1$:*

$$\frac{y}{y^*} = 1 - [\exp(-\xi t/\tau)]\{\cosh[(t/\tau)(\sqrt{(\xi^2-1)}] - [\xi/(\sqrt{(\xi^2-1)})][\sinh((t/\tau)(\sqrt{(\xi^2-1)})]\} \tag{8}$$

For any values of $\xi$ and $\tau$, Eqs. 6 to 8 can be used accurately to calculate the specified response. The values of $\xi$ and $\tau$, which correspond to the desired specified response, are selected, and the generic control loop constants are computed as $k_1 = 2\xi/\tau$ and $k_2 = 1/\tau^2$.

## Robust Generic Model Control

Any errors in modeling the process are intended to be compensated for by the integral action of the controller. However, for a critically damped or overdamped specification, $k_2$ is negligible compared to $k_1$. As a result, the integral action is reduced to the point where offset elimination is extremely sluggish.

A robust generic model control (RGMC) structure is proposed (Lundberg and Bezanson, 1990), in which Eq. 4 is augmented with the error in output derivative:

$$\dot{g}\,(x) = r^* - [\dot{y}\,(t) - \dot{y}_m(t)] \tag{9}$$

where $r^*$ is defined by Eq. 3. Lundberg and Bezanson (1990)

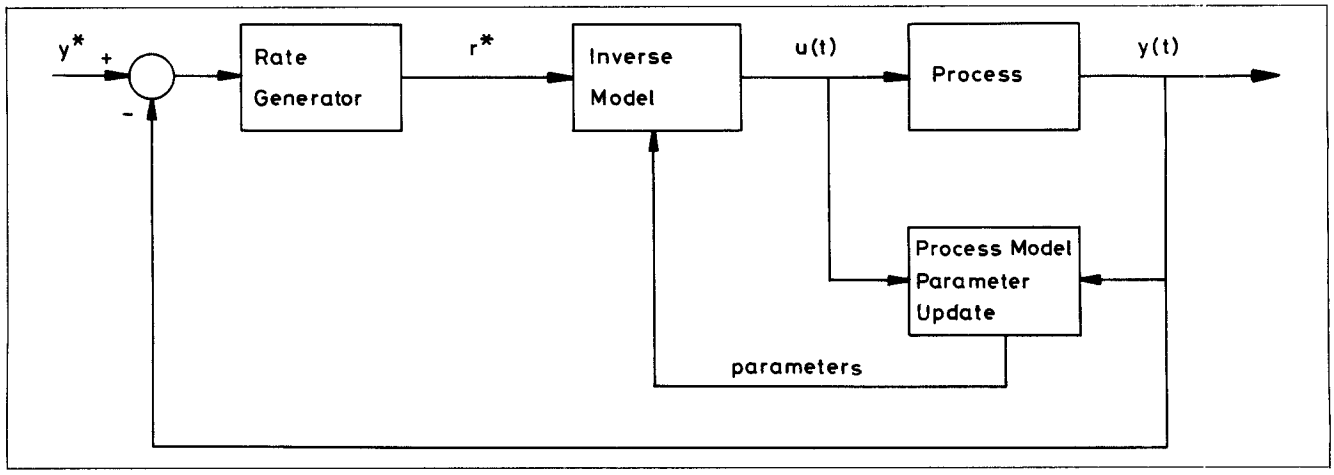


**Figure 1. Adaptive generic model control structure.**

have reported GMC and RGMC structures. RGMC is represented as GMC with modeling error compensation. The process output asymptotically approaches the set point and no offset occurs for any value of $k_2$.

## Adaptive Generic Model Control

There are two ways of correcting the process model output for the process/model mismatch: 1. a correction or error term can be added to the process model output explicitly and 2. the process model parameters can be updated implicitly so that the model output matches with the process output. In robust generic model control, the process/model mismatch is compensated with the help of the first alternative by adding the correction term to the model output time derivative.

RGMC considers that the real process and process model are represented by the same equation structure, Eqs. 1 and 2. Therefore, RGMC can take care of change in parameters. But, it is not always possible to represent the process model exactly as real process fundamental equations, because all the process variables are not measurable in chemical processes. The process model, therefore, is assumed to have a different structure from the process, so that only limited information about the process measurements is necessary:

$$\dot{y}_m = h(y,u) \tag{10}$$

With these limitations, RGMC is unable to cope with process/model mismatch.

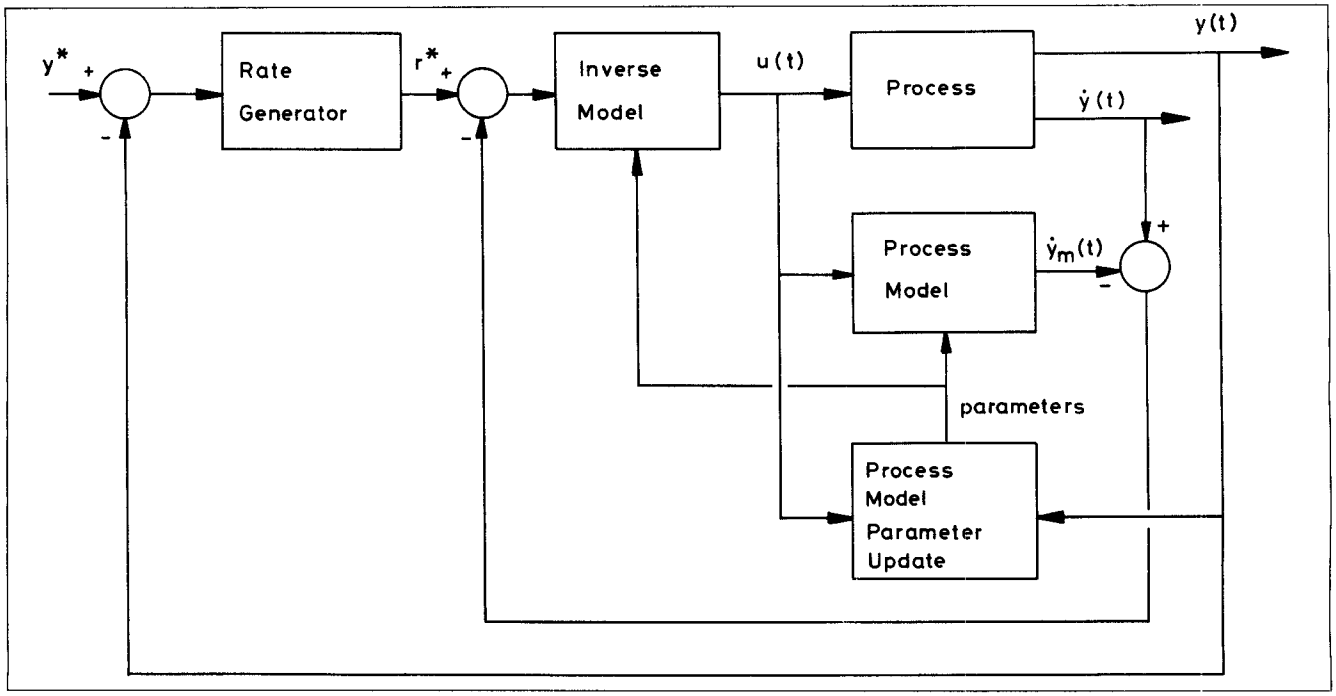


**Figure 2. Adaptive robust generic model control structure.**

In this study, an adaptive generic model control structure is proposed, Figure 1. The process/model mismatch is compensated by the second alternative mentioned earlier in this control structure, which updates the model parameters on-line and uses the available input and the output measurements of the process. Information about these updated parameters or process model is used by the generic model controller in generating the manipulated input signals to the process.

Adaptive GMC is similar to model reference adaptive control in that adaptive GMC has a reference output rate profile ($r^*$), whereas model reference adaptive control has a reference model to predict the output. Adaptive GMC also is similar to self-tuning adaptive control in that both of these approaches incorporate updating of process model parameters. It, however, differs from both self-tuning and model reference adaptive controllers because controller settings are finally being updated in these adaptive controllers. On the other hand, the generic control-loop constants (which are the controller settings) are constant, but the updated model that compensates for process/model mismatch is used directly in the control law of adaptive GMC.

## Adaptive Robust Generic Model Control

In the previous section, GMC has been extended to adaptive GMC. On similar lines, RGMC is extended further to adaptive RGMC, Figure 2: adaptive RGMC includes the model parameter updating in addition to the RGMC structure.

### *Example 1*

Consider a nonlinear process,

$$\dot{y} = ay^2 + bu + cuy \tag{11}$$

that has been modeled as:

$$\dot{y}_m = \hat{a}y^2 + \hat{b}u + \hat{c}uy \tag{12}$$

where $\hat{a} = -0.25$, $\hat{b} = 0.50$, and $\hat{c} = 1.75$ are perfect model parameters. Suppose that there is $-50\%$ parametric mismatch in $c$: $a = \hat{a}$, $b = \hat{b}$ and $c = 0.5\,\hat{c}$.

### *Example 2*

The nonlinear process, Eq. 11, is modeled as the nonlinear model, Eq. 12, with $+50\%$ parametric mismatch in $c$: $a = \hat{a}$, $b = \hat{b}$, and $c = 1.5\,\hat{c}$.

### *Example 3*

The nonlinear process, Eq. 11, has been approximated as a linear model:

$$\dot{y}_m = \hat{a}y + \hat{b}u \tag{13}$$

with $\hat{a} = 0.1764$ and $\hat{b} = 0.4225$. There is a structural mismatch between the process and the model.

### *Example 4*

In this example, the nonlinear process described by Eq. 11 is modeled as:

$$\dot{y}_m = \hat{a}y^2 + \hat{b}u \tag{14}$$

with $\hat{a} = 0.6413$ and $\hat{b} = 0.5496$. Here also, there is a structural process/model mismatch, but the model is nonlinear unlike the previous example.

## Results and Discussion of Examples

Examples 1 to 4 are worked out with overdamped and underdamped closed-loop output specifications: $k_1 = 0.25$ and $k_2 = 0.0001$ for overdamped specification and $k_1 = 0.25$ and $k_2 = 0.1$ for underdamped specification. The sampling interval used in these examples is 0.5 time units. The results of these examples are plotted in Figures 3 to 6 with *a* referring to overdamped response and *b* referring to underdamped specification.

Lee and Sullivan (1988) and Lundberg and Bezanson (1990), while advocating GMC and RGMC, respectively, did not plot the specified response or reference trajectory along with the obtained response. The specified response is also shown along with GMC, RGMC, AGMC and ARGMC responses in all the example figures. The specified response is calculated using Eqs. 6 and 8 for the above-mentioned $k_1$ and $k_2$ values, instead of using the figure reported by Lee and Sullivan (1988).

Parameter estimation of a dynamic model via least squares

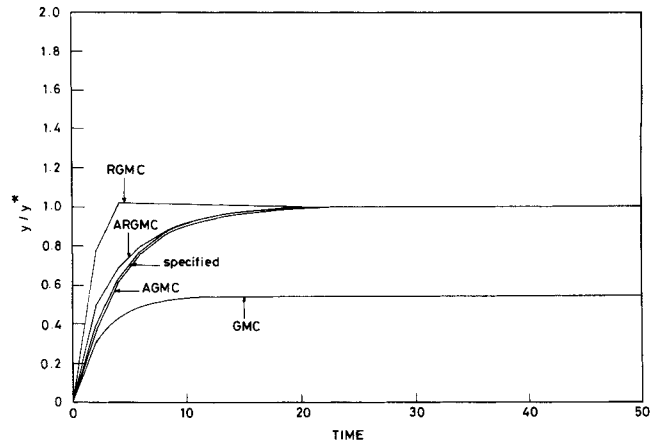


**Figure 3a. Overdamped response of example 1.**

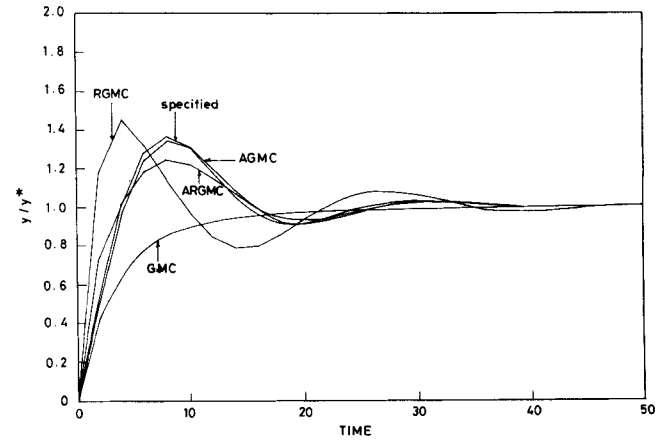


**Figure 3b. Underdamped response of example 1.**

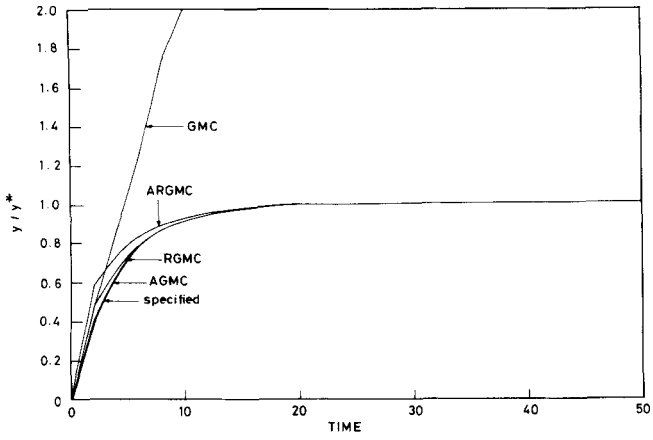


**Figure 4a. Overdamped response of example 2.**

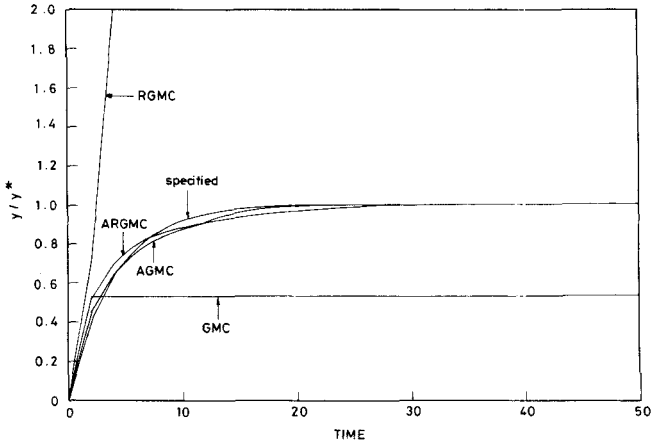


**Figure 5a. Overdamped response of example 3.**

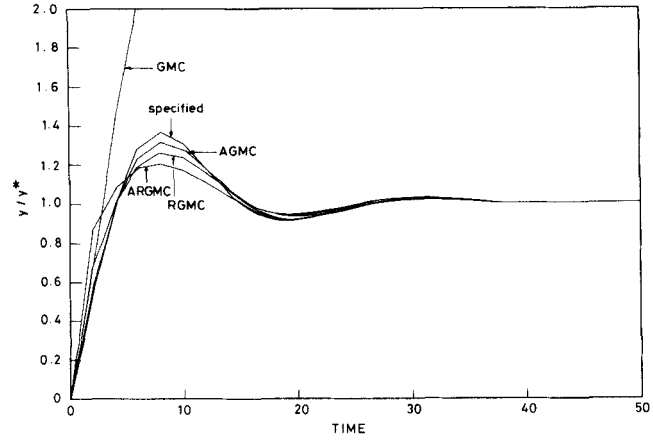


**Figure 4b. Underdamped response of example 2.**

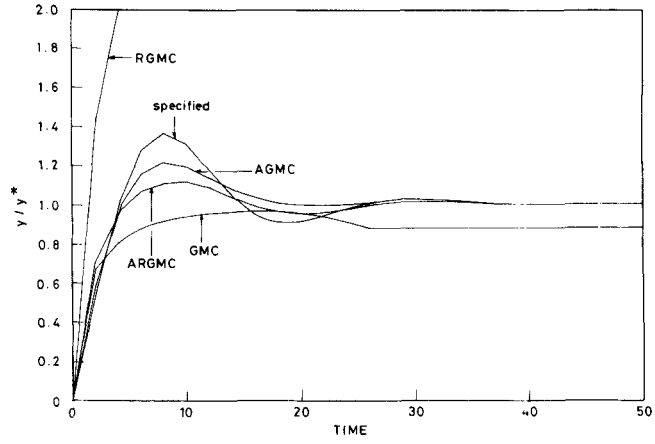


**Figure 5b. Underdamped response of example 3.**

is the simplest scheme available. Its handy, recursive form makes it very appealing for on-line adaptation. For the adaptation in AGMC and ARGMC to be implementable on-line, "one-shot" algorithm is employed for the initial parameters and covariance estimation. The following recursive least squares formula, the Hsia algorithm (Hsia, 1977; Rolf and Lim, 1984), is used for on-line estimation of parameters and covariance matrix after each new sample:

$$\theta(k+1)=\theta(k)+\gamma(k+1)P(k) \times v(k+1)[y(k+1)-v^T(k+1)\theta(k)] \quad (15)$$

$$P(k+1)=\frac{1}{\lambda}[P(k)-\gamma(k+1)P(k) \times v(k+1)v^T(k+1)P(k)] \quad (16)$$

$$\gamma(k+1)=1/[1+v^T(k+1)P(k)v(k+1)] \quad (17)$$

where $\theta$ is the parameter vector, $\gamma$ is the intermediate estimation variable, $P$ is the covariance matrix, $v$ is the vector of input-output variables, $y$ is the output variable, and $0<\lambda\leq 1$ is the forgetting factor.

When linear and nonlinear processes are modeled perfectly, all the four control algorithms—GMC, RGMC, AGMC, and ARGMC—produce results that are very close to the specified response. For the two examples reported by Lundberg and Bezanson (1990), both AGMC and ARGMC result in responses almost equivalent to RGMC, whereas the GMC response exhibits offset in the overdamped cases.

Lee et al. (1989b) defined structural mismatch in a linear transfer function process as the mismatch in order between the process transfer function and model transfer function, whereas the structural mismatch in this study refers to the mismatch in the nonlinearity of the process and the state-space model. Further, in the process model mismatch compensation algorithm of Lee et al. (1989b), three additional design parameters are to be specified (especially the filter parameter specification is crucial), whereas the RGMC algorithm (Lundberg and Bezanson, 1990), while not requiring any additional design parameters, can compensate for linear as well as nonlinear parametric mismatch.

The results of examples 1 and 2 are plotted in Figures 3a, 3b, 4a and 4b. In these examples, the nonlinear process is modeled as a nonlinear model with the parametric mismatch. Figure 3a indicates that GMC results in offset, whereas Figure 4a shows that GMC leads the process to instability. Figures 3a and 4a also show that the specified response and the AGMC response almost coincide, while there is a difference between the specified response, and RGMC and ARGMC. The same

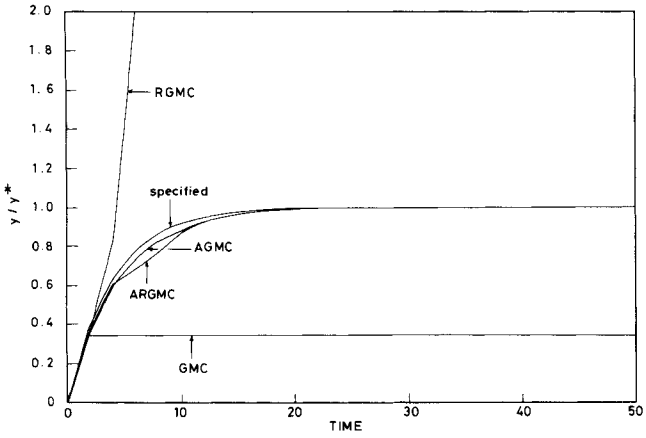


**Figure 6a. Overdamped response of example 4.**

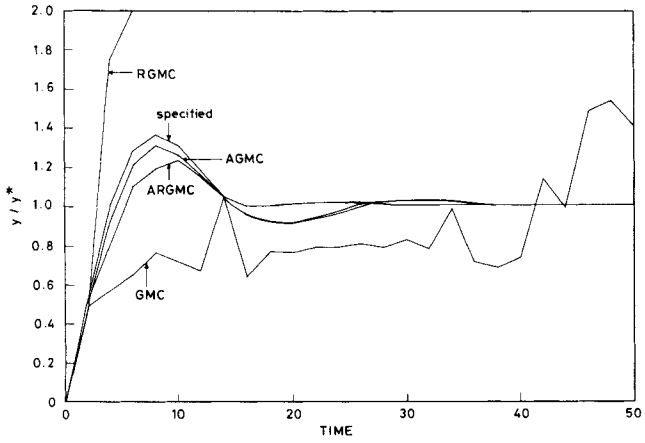


**Figure 6b. Underdamped response of example 4.**

features are observed in Figures 3b and 4b except that GMC is also able to reach the desired set point in Figure 3b, but the response obtained is overdamped when the specified response is underdamped.

Figures 5a, 5b, 6a and 6b exhibit the responses of examples 3 and 4. These are the cases of structural mismatch of a nonlinear process. Figures 5a and 5b show the results of a nonlinear process modeled as a linear model, whereas Figures 6a and 6b are the results of a nonlinear process modeled nonlinearly, but with a structural mismatch. These four figures clearly indicate that RGMC leads the process to instability with both overdamped and underdamped output specifications. In the overdamped case, Figures 5a and 6a, GMC produces steady-state offset, whereas in the underdamped case of Figures 5b and 6b, the GMC response is irregularly oscillatory. All the four figures indicate that both AGMC and ARGMC are able to follow the specified response closely, but as observed in the other figures, AGMC behaves in a better way than ARGMC.

When there is no adaptation, robust generic model control exhibits better performance than generic model control, provided that there is no structural mismatch. RGMC takes care of parametric mismatch well. With structural mismatch, both GMC and RGMC fail to produce satisfactory results. To take care of structural mismatches, in this study, AGMC and ARGMC are proposed. These algorithms have produced good results in the cases of parametric as well as structural process/model mismatches. When GMC, RGMC, AGMC, and ARGMC are compared, AGMC exhibits better performance in all the cases of no process/model mismatch, parametric mismatch as well as structural mismatch.

## Distillation Adaptive Generic Model Control

Control of the top and bottom compositions of a distillation can be a difficult task due to the presence of control-loop interactions and nonlinearities. Dramatic increases in energy costs have sharpened the incentive to improve control to achieve the desired or improved separation at lower cost through less energy usage. Consequently, considerable effort is being directed to finding control strategies that will give better control performance than can be obtained through the use of conventional two PI or PID controllers, in which typically one or both of the controllers must be conservatively tuned to achieve a satisfactory degree of stability. However, conservative tuning results in several undesirable effects such as long response times and large deviations after load disturbances.

The difficulty involved in suitable control of a distillation column steps from nonlinear column dynamic behavior which is asymmetric in nature. Consequently, controllers may need frequent retuning to compensate for changes in column dynamics as operating conditions change. Thus, an ideal controller would be one that is able to adapt its parameters to compensate for these changes. Adaptive control, usually based on simultaneous model identification and control, is normally employed when the actual process is nonlinear and approximate models are used for the purposes of implementation.

In this study, yet another objective is to develop a generic model controller for dual composition control of distillation. Generic model control requires the imbedding of state-space dynamic equations of the distillation process into the control law. That is, the controller is to be formulated by solving the dynamic process model for the derivative of the controlled variables and letting them equal to what is, in effect, a proportional integral term operating on the difference between the current value of the controlled variables and their desired set values. A *nt*-staged distillation column is described by $(3nt+6)$ differential equations and many algebraic equations (Luyben, 1990). Hence, GMC described by Lee and Sullivan (1988) and Lundberg and Bezanson (1990) cannot be applied to distillation process. Moreover, in practice, most of the variables encountered in model equations are not measured. Therefore, distillation column, along with condenser and reboiler, is considered as a black box, which is simple to handle and requires the information of only input and output variables, with terminal compositions as process outputs, and reboiler heat duty and reflux rates as process inputs, Figure 7.

Instead of $(3nt+6)$ state-space equations, only two differential equations of $x_D$ and $x_B$ are considered in this study using structurally simplified linear and nonlinear models. This approximation is valid only when the system is very close to the normal operating conditions. To make the two differential equations applicable over a wide range of operating conditions, on-line adaptation of the black box model parameters becomes necessary. Distillation adaptive generic model control in Figure 8 is proposed using black box models due to the better performance of AGMC over GMC, RGMC, and ARGMC.

Two types of black box models are proposed for on-line adaptation of distillation: linear and nonlinear.

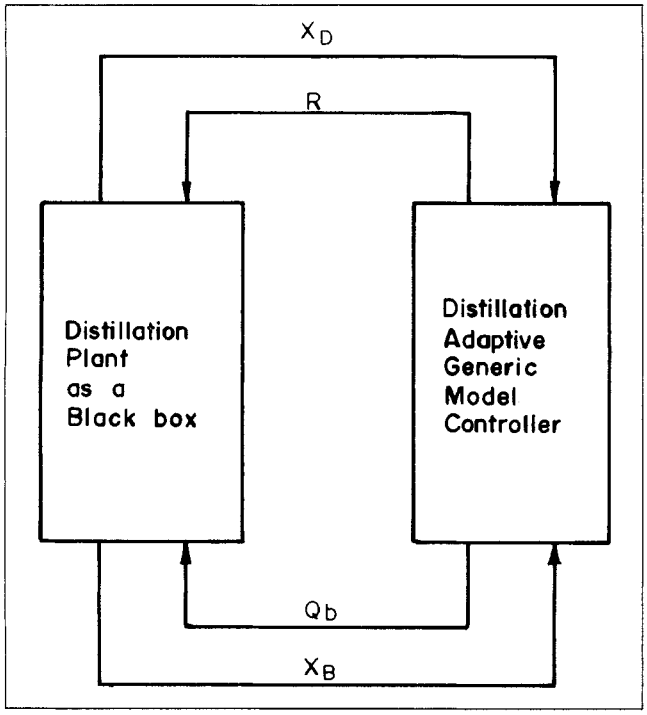


**Figure 7. Control of a distillation plant.**

### *DAGMC using linear black box model*

The linear black box model considered for on-line adaptation is:

$$x_D(k+1)=a_{11}x_D(k)+a_{12}x_B(k)+b_{11}R(k)+b_{12}Q_b(k) \quad (18)$$

$$x_B(k+1)=a_{21}x_D(k)+a_{22}x_B(k)+b_{21}R(k)+b_{22}Q_b(k) \quad (19)$$

These equations can also be written as a single equation using vector matrix notation;

$$\begin{bmatrix} x_D(k+1) \\ x_B(k+1) \end{bmatrix} = \begin{bmatrix} a_{11}a_{12}b_{11}b_{12} \\ a_{21}a_{22}b_{21}b_{22} \end{bmatrix} \begin{bmatrix} x_D(k) \\ x_B(k) \\ R(k) \\ Q_b(k) \end{bmatrix} \quad (20)$$

Further, $x_D(k+1)$ and $x_B(k+1)$ can be obtained from the Taylor series expansion around $x_D(k)$ and $x_B(k)$ with higher order terms neglected:

$$x_D(k+1)=x_D(k)+Tdx_D(k)/dt \quad (21)$$

$$x_B(k+1)=x_B(k)+Tdx_B(k)/dt \quad (22)$$

Equating Eqs. 18 and 21, and Eqs. 19 and 22 provides the differential equations for the controlled variables, which are required for DAGMC.

$$\dot{x}_D(k)=dx_D(k)/dt=((a_{11}-1)x_D(k)+a_{12}x_B(k)+b_{11}R(k)+b_{12}Q_b(k))/T \quad (23)$$

$$\dot{x}_B(k)=dx_B(k)/dt=(a_{21}x_D(k)+(a_{22}-1)x_B(k)+b_{21}R(k)+b_{22}Q_b(k))/T \quad (24)$$

The reference rate for the process is generated from the setpoint deviation as:

$$r_D^*=\dot{x}_D^*=k_{11}(x_D^*-x_D)+k_{12}\int(x_D^*-x_D)dt \quad (25)$$

$$r_B^*=\dot{x}_B^*=k_{21}(x_B^*-x_B)+k_{22}\int(x_B^*-x_B)dt \quad (26)$$

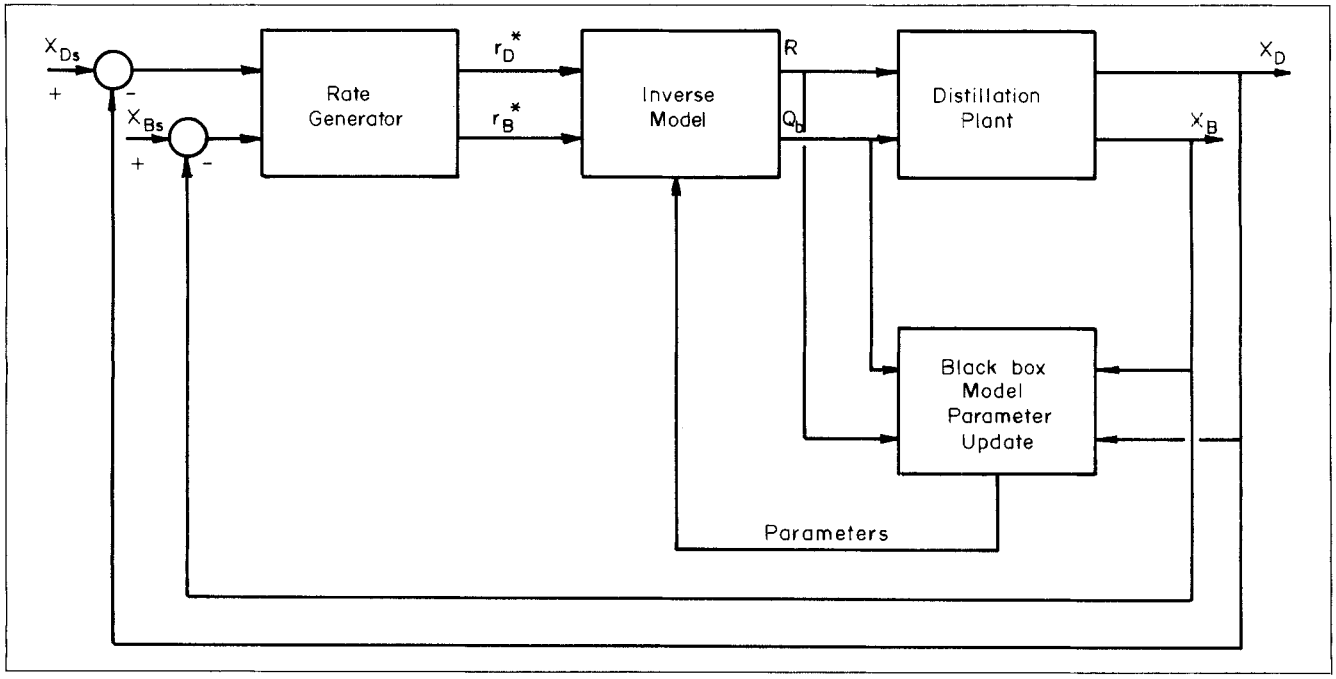


**Figure 8. Distillation adaptive generic model control structure.**

where $r_D^*$ and $r_B^*$ are the desired process output rates, $x_D^*$ and $x_B^*$ are the setpoints, and $k_{11}$, $k_{12}$, $k_{21}$ and $k_{22}$ are generic control-loop constants. The control signals are obtained by setting:

$$r_D^* = \dot{x}_D \tag{27}$$

$$r_B^* = \dot{x}_B \tag{28}$$

and solving for $R$ and $Q_b$:

$$\begin{bmatrix} R \\ Q_b \end{bmatrix} = \begin{bmatrix} b_{11}b_{12} \\ b_{21}b_{22} \end{bmatrix}^{-1} \left[ T \begin{bmatrix} k_{11}(x_D^* - x_D) + k_{12}\int (x_D^* - x_D)dt \\ k_{21}(x_B^* - x_B) + k_{22}\int (x_B^* - x_B)dt \end{bmatrix} - \begin{bmatrix} a_{11}-1 & a_{12} \\ a_{21} & a_{22}-1 \end{bmatrix} \begin{bmatrix} x_D \\ x_B \end{bmatrix} \right] \tag{29}$$

### *DAGMC using nonlinear black box model*

The top and bottom composition dynamics are described by:

$$\dot{x}_D = [V_{nt}y_{nt} - (R+D)x_D]/M_D \tag{30}$$

$$\dot{x}_B = (L_1x_1 - V_By_B - Bx_B)/M_B \tag{31}$$

where

$$V_B = (Q_b - L_1(h_B - h_1))/(H_B - h_B) \tag{32}$$

Substitution of Eq. 32 in Eq. 31 results in:

$$\dot{x}_B = (L_1x_1 - (y_B(Q_b - L_1(h_B - h_1))/(H_B - h_B)) - Bx_B)/M_B \tag{33}$$

Eqs. 30 and 33 are further simplified as:

$$\dot{x}_D = p_{11}x_D + p_{12}Rx_D + p_{13} \tag{34}$$

$$\dot{x}_B = p_{21}x_B + p_{22}Q_b + p_{23} \tag{35}$$

where

$$p_{11} = -D/M_D;\ p_{12} = -1/M_D;\ p_{13} = V_{nt}y_{nt}/M_D;$$

$$p_{21} = -B/M_B;\ p_{22} = -y_B/[M_B(H_B - h_B)];$$

$$p_{23} = \{(L_1x_1/M_B) + [L_1(h_B - h_1)y_B]/[M_B(H_B - h_B)]\}$$

The reference rate for the process is generated from the setpoint deviations according to Eqs. 25 and 26. The control signals are derived according to Eqs. 27 and 28 and by solving for $R$ and $Q_b$:

$$R = [k_{11}(x_D^* - x_D) + k_{12}\int (x_D^* - x_D)dt - p_{13} - p_{11}x_D]/(p_{12}x_D) \tag{36}$$

$$Q_b = [k_{21}(x_B^* - x_B) + k_{22}\int (x_B^* - x_B)dt - p_{23} - p_{21}x_B]/p_{22} \tag{37}$$

Equations 34 and 35 are substituted into Eqs. 21 and 22, which results in:

$$x_D(k+1) = q_{11}x_D(k) + q_{12}Rx_D(k) + q_{13} \tag{38}$$

$$x_B(k+1) = q_{21}x_B(k) + q_{22}Q_b(k) + q_{23} \tag{39}$$

where

$$q_{11} = p_{11}T + 1;\ q_{12} = p_{12}T;\ q_{13} = p_{13}T;$$

$$q_{21} = p_{21}T + 1;\ q_{22} = p_{22}T;\ q_{23} = p_{23}T$$

The parameters of Eqs. 38 and 39 are adapted on-line similar to those of Eqs. 18 and 19.

The Hsia algorithm, Eqs. 15 to 17 (Hsia, 1977; Rolf and Lim, 1984), is used for on-line adaptation of distillation model parameters. Equations 38 and 39 cannot be written in the matrix vector form like Eq. 20. Therefore, while one covariance matrix of dimension $4 \times 4$ is sufficient for on-line adaptation of linear model Eq. 20, two covariance matrices of dimension $3 \times 3$ are necessary to update the parameters of the two nonlinear model, Eqs. 38 and 39.

## Systems Studied

For evaluation of distillation control schemes, models for process dynamics are needed. Realistic performance of an actual column seldom can be predicted satisfactorily by excluding simultaneous effects of heat transfer and fluid flow on plates. Plate hydraulics is an important factor in predicting the dynamic performance. Because of its effects on heat and mass transfer calculations, accurate predictions of physical properties are also important. Modeling from first principles resulted in rigorous nonlinear, dynamic material and component,

**Table 1. Details of Distillation Columns**

| | Methanol-Water Column | Debutanizer Column |
|---|---|---|
| *Number of Trays* | 8 | 8 |
| *Feed Plate Position* | 4 | 5 |
| *Column Pressure (atm)* | 1 | 6.67 |
| *Components* | Methanol-Water | *i*-Butane *n*-Pentane |
| *Feed Properties* | | |
| Composition (more volatile mole fraction) | 0.33 | 0.5 |
| Flow rate (kmol/h) | 2.953 | 113.4 |
| Temperature (K) | 348.556 | 339.98 |
| *Distillate Properties* | | |
| Composition (more volatile mole fraction) | 0.93309 | 0.9955 |
| Flow rate (kmol/h) | 1.036 | 16.376 |
| Temperature (K) | 339.68 | 323.344 |
| *Bottom Product Properties* | | |
| Composition (more volatile mole fraction) | 0.0040137 | 0.416 |
| Flow rate (kmol/h) | 1.917 | 97.023 |
| Temperature (K) | 372.232 | 347.902 |
| *Steady-State Operating Conditions* | | |
| Reboiler heat duty (kJ/h) | 1.028E+05 | 2.005E+06 |
| Reflux rate (kmol/h) | 1.71 | 68 |

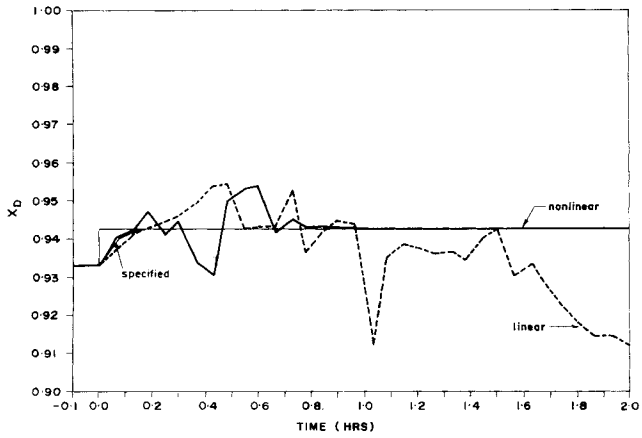


**Figure 9. $x_D$ for +1% setpoint change in top composition: methanol-water column.**

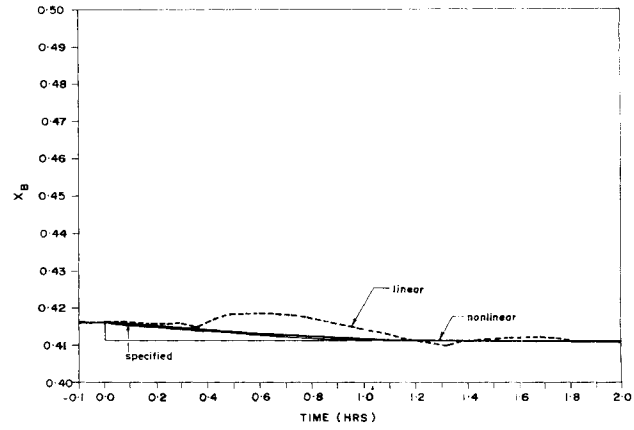


**Figure 11. $x_B$ for −1% setpoint change in bottom composition: debutanizer.**

and algebraic energy equations with vapor-liquid equilibria. The distillation dynamics package has major computation subroutines of tray hydraulics, enthalpy calculations, average molecular weight and density calculations and vapor-liquid equilibria.

The first distillation process examined to illustrate DAGMC is methanol-water distillation column (Wood and Berry, 1973). The properties of feed, distillate and bottom streams along with the steady-state operating conditions chosen for this distillation column are given in Table 1.

The second distillation process studied is a debutanizer column (Gani et al., 1986; Ruiz et al., 1988) which split a feed stream of isobutane and pentane entering the column at its bubble point. In Table 1, the steady-state operating conditions are given along with the properties of feed, distillate and bottom streams. Full details of this column are given by Gani et al. (1986) and Ruiz et al. (1988).

## Results and Discussion of Distillation Systems

The top and bottom composition responses to setpoint and load disturbances are shown in Figures 9 and 10 for methanol-water column and in Figures 11 and 12 for the debutanizer column. In these figures, the ordinate scale is magnified to demonstrate the effectiveness of linear and nonlinear DAGMC algorithms. The sampling interval in both the columns is 0.01 h. The generic control-loop constants, $k_{11}$ ($h^{-1}$), $k_{12}$ ($h^{-2}$), $k_{21}$ ($h^{-1}$) and $k_{22}$ ($h^{-2}$), used in this study for the methanol-water column are 17.24, 8.26, 20.57, and 11.76, respectively. For the debutanizer column, the generic control-loop constants are 1.3, 1.69, 1.3, and 1.69, respectively.

Figure 9 exhibits the performance of the linear and nonlinear DAGMC, when a +1% setpoint disturbance in the top composition is introduced into the methanol-water column. This figure indicates the superiority of nonlinear DAGMC over linear DAGMC. This figure also shows the much degraded performance of linear DAGMC. Similar performance is observed in $x_B$ response.

Figure 10 shows the response to +10% load disturbance in the feed flow rate to methanol-water column. This figure indicates that linear DAGMC results in offset in $x_D$, and the controller is unable to nullify the effect of load disturbance, whereas nonlinear DAGMC has resulted in a better response. The same features are observed in the $x_B$ response also.

Figure 11 shows the bottom terminal composition response of debutanizer column to −1% setpoint disturbance in the bottom composition. The top composition is perfectly maintained at its setpoint using both linear and nonlinear DAGMC.

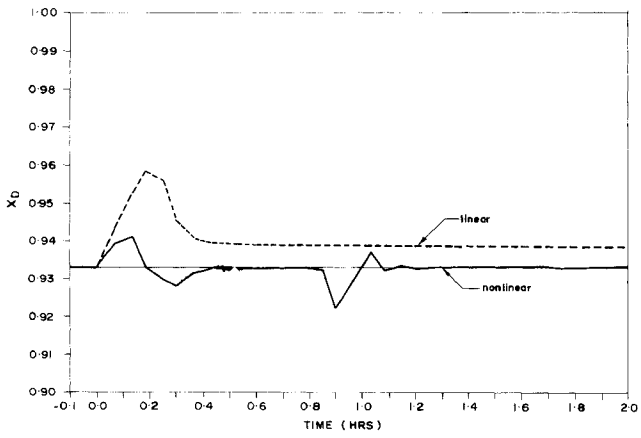


**Figure 10. $x_D$ for +10% load disturbance in feed flow rate: methanol-water column.**

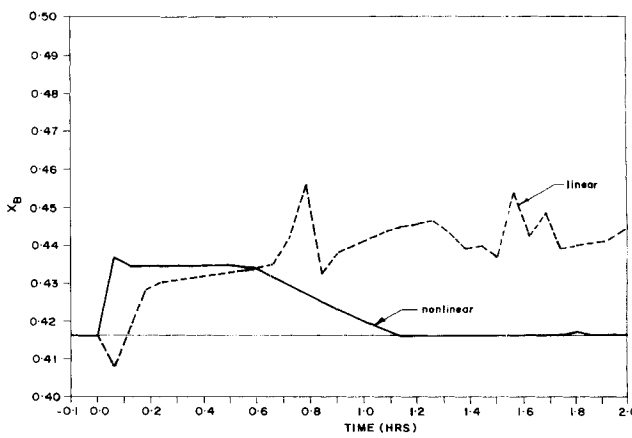


**Figure 12. $x_B$ for +10% load disturbance in feed flow rate: debutanizer.**

The bottom composition is smoothly changed to the new setpoint with nonlinear DAGMC, whereas there is a little overshoot using linear DAGMC.

Figure 12 illustrates the behavior of debutanizer column to +10% load disturbance in the feed flow rate. As observed previously, the response of the top composition is controlled perfectly with linear as well as nonlinear DAGMC, whereas the response of the bottom composition with linear DAGMC is not satisfactory, because it has both oscillations and offset. This figure also indicates the improved performance of nonlinear DAGMC.

In distillation control, using the real state-space model as the *base case*, is not *practical*, whereas in a simulation situation a good basis for comparison can be the hypothetical base case. Therefore, hypothetical base-case generic model control is applied to distillation columns with the earlier reported generic control-loop constants using a real state-space model and its results compared for setpoint disturbances with those of nonlinear DAGMC. It is observed that for the same sampling interval used with nonlinear DAGMC algorithm, the hypothetical base-case GMC responses are very sluggish and produce offset. The hypothetical base-case RGMC responses also are similar to GMC responses. Recently, Brown et al. (1990) have stated that if there is no process/model mismatch and a sufficiently small sampling interval is chosen, the GMC closed-loop output response will follow exactly the reference trajectory or specified response. In the hypothetical base-case of GMC application to distillation, since there is no process/model mismatch, the sampling interval is reduced by a factor of 10 to ascertain its effect. The hypothetical base-case GMC response with a reduced sampling interval is almost the same as the nonlinear DAGMC response with no reduction in sampling interval. The hypothetical base-case studies do not find any practical utility except that these studies confirm the statement of Brown et al. (1990) and also indicate that nonlinear distillation adaptive generic model control can accommodate larger sampling interval due to its implicit adaptation.

Since imbedding of the distillation state-space model in the generic model control law is almost impossible, only two terminal composition equations are employed. In linear-model-based DAGMC, two linear difference equations, Eqs. 18 and 19, are used with adaptation, whereas in nonlinear model-based DAGMC, the actual nonlinear state-space top and bottom composition equations, Eqs. 34 and 35, are employed with adaptation. Therefore, nonlinear DAGMC has a better theoretical basis than linear DAGMC. The results also demonstrate that nonlinear distillation adaptive generic model control is efficient in controlling distillation processes.

## Conclusions

The proposed AGMC and ARGMC structures are applied along with GMC and RGMC to four examples for a setpoint change with overdamped and underdamped closed-loop specifications. AGMC has exhibited better performance over ARGMC, GMC, and RGMC in all the cases of no process/model mismatch, parametric mismatch, and structural mismatch.

The proposed linear and nonlinear distillation adaptive generic model control algorithms are applied to two typical distillation columns: methanol–water column and debutanizer column. Nonlinear DAGMC has exhibited better performance over linear DAGMC in the case of setpoint and load disturbances.


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## Notation

$a, b, c, p, q$ = parameters
$\hat{a}, \hat{b}, \hat{c}$ = process model parameters
$B$ = bottom product flow rate
$D$ = distillate flow rate
$f, g$ = arbitrary functions
$F$ = feed flow rate
$h$ = liquid enthalpy and arbitrary function
$H$ = vapor enthalpy
$k_1, k_2, k_{11}, k_{12}, k_{21}, k_{22}$ = generic control-loop constants
$L$ = liquid flow rate
$M$ = molar holdup
$P$ = covariance matrix
$Q$ = heat duty
$r^*$ = reference rate
$R$ = reflux rate
$s$ = Laplace transform variable
$t$ = time variable
$T$ = sampling interval
$u$ = process input
$v$ = vector of input-output variables
$V$ = vapor flow rate
$x$ = system state variable or liquid composition (mol fraction)
$y$ = system output or vapor composition (mol fraction)
$Y(s)$ = Laplace transform of process output

### *Greek letters*

$\gamma$ = intermediate estimation variable
$\lambda$ = forgetting factor
$\xi, \tau$ = constants
$\theta$ = parameter vector

### *Subscripts*

$b$ = reboiler
$B$ = bottom product
$D$ = top product
$m$ = model
$nt$ = number of trays

### *Superscript*

$*$ = setpoint

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